ALTA
Institute for Algebra, Geometry, Topology and their Applications

Prof. Dr. Dmitry Feichtner-Kozlov
Discrete Mathematics
WS 2025/26

Exercise Set 2

Exercise 2.1 Show that for any nonnegative integer n, and any positive integer t, we have

$$\sum_{k_1,\dots,k_t} \binom{n}{k_1,\dots,k_t} = t^n, \tag{1}$$

where the sum is taken over all t-tuples of nonnegative integers $k_1,\dots,k_t$, such that $k_1+\dots+k_t=n$. (6 Punkte)

Exercise 2.2 Show that for any nonnegative integer n, and any positive integer t, we have

$$\sum_{k_1,\dots,k_t} (-1)^{k_2+k_4+k_6+\dots} \binom{n}{k_1,\dots,k_t} = \begin{cases} 0, & \text{if } t \text{ is even,} \\ 1, & \text{if } t \text{ is odd.} \end{cases} \tag{2}$$

where the sum is taken over all t-tuples of nonnegative integers $k_1,\dots,k_t$, such that $k_1+\dots+k_t=n$.

(6 Punkte)

Exercise 2.3 Show the following identity for $n \geq 0$:

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k}^2 = \begin{cases} (-1)^{n/2}\binom{n}{n/2}, & \text{if } n \text{ is even;} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

(6 Punkte)

Exercise 2.4 Let k and N be positive integers. Show that there exists a unique integer $1 \leq m \leq k$, and unique integers $x_m,\dots,x_k$, such that $x_k > x_{k-1} > \dots > x_m \geq m$, and we have the decomposition

$$N = \binom{x_k}{k} + \binom{x_{k-1}}{k-1} + \dots + \binom{x_m}{m}. \tag{4}$$

(6 Punkte)

Abgabe der Hausübungen: Di, 28.10.25, vor dem Tutorium (Di, 12:15 Uhr) in das Postfach 54 in MZH 1. Ebene, oder Abgabe zu Beginn des Tutoriums um 12:30 Uhr